COURSE SCHEDULING TO FIND THE
MINIMUM COST SET OF FACILITIES REQUIRED

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THESIS

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by

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ABSTRACT

The problem of determining the quantity of classrooms, laboratories and instructors to train sections of students attending numerous distinct courses in a school such as the Fleet Ballistic Missile School is considered.

A procedure is developed for determining feasible schedules in order to graduate a fixed number of trainees over time while minimizing the cost of facilities mix required.

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I. INTRODUCTION

The problem of determining the capacity of a school to train sections of students attending numerous distinct courses has been considered by Willingham [1] as an optimization problem. He used linear programming techniques for determining the maximum number of convenings of each type of course taught at the school which can take place during a year period subject to resource constraints and lower bounds on the number of convenings of each type of course.

This problem arises with the conversion of FBM submarines from Polaris to Poseidon missile systems. There exists the necessity of assuring a proper quantity and mix of resources to instruct all required personnel at the Naval Guided Missile School. Here the primary resources of interest are laboratory facilities of various types with their associated equipment, classrooms and instructors of each specialty pertinent to Poseidon technical training [1].

The problem as stated from the point of view of those responsible for the funding and operation of the school is: "Given the planning estimates from BUPERS of the number of personnel who require taining in each type of course over a specified time period, determine what level and mix of resources is adequate to carry out the mission." The present work will consider this last aspect of the problem,

attacking it under the assumption that carefully scheduling the convenings of each section will yield to a minimum requirement of resources for a given time period.

The above idea can be expressed more precisely and in general as follows: An important function of any school planning is to estimate the number of facilities, namely laboratories, classrooms and instructors, that are needed to support a specified program of instruction and to specify, in general terms, the schedule to obtain the best employment of the resources. By a specified program we mean the type of courses to be taught, their curricular requirements and the quantity of students assigned to each one. The problem is, then, to find that schedule which requires the least cost set of facilities.

Since a particular schedule dictates a required set of facilities, it is necessary to have a measure of some characteristic in order to be able to compare it to other feasible schedules. A measure of effectiveness of a schedule will be defined as the cost involved in the installation of all facilities associated with it.

In general, specifying a measure of effectiveness in a scheduling problem is specifying a set of equivalence classes of schedules and a preference ordering among these classes. For example, specifying the cost of the facilities needed as the measure of effectiveness for a particular problem means that:

(a) All schedules that have a cost C_o are equivalent, so

one is indifferent as to which schedule is selected.

(b) A schedule with cost C_1 is preferred to a schedule with cost C_2 if and only if C_1 is strictly less than C_2.

From the above discussion it can be seen then, that the minimal cost scheduling problem is one of evaluating schedules and the associated combination of resources in the search for the minimum cost. It is clear that there is a very large number of possible schedules for the problem. It is also clear that most such schedules are uninteresting for reasonable measures of performance and only a few are worth considering.

Although the least cost facility mix scheduling problem is easy to state and to visualize, no easy solution is available. Evaluating the set of all feasible solutions (represented by all possible combinations of schedules and facilities) is, for practical purposes, a gigantic quest. The objective of this paper is to illustrate a method of solving the problem applying the concepts developed in [2]. A procedure will be presented for approaching one schedule of the equivalence class of schedules that requires the least cost of the resources for a given program of instruction for a given time period.

A first attempt to obtain an acceptable solution to the scheduling problem can be made by evaluating the costs of random schedules which, consequently are random variables too, estimating the distribution of the costs, and then attempting to estimate the probability that, in a sample of

size n, at least one will be in the lowest fraction P_ℓ of the population. For a discrete random variable, as will be the case here, let p_r be the probability that a single observation has a cost value equal to r. Then the probability that the smallest member of a sample of size n has a cost value less than r is given by:

$$P[Z_{min}<r] = 1 - (1-P_\ell)^n \approx 1-e^{-nP_\ell}$$

$$\text{where} \quad P_\ell = \sum_{i=1}^{r-1} p_i, \qquad Z_i<r.$$

For obvious reasons it is desirable to have an estimate of the lowest cost which is as close as possible to the true minimum cost of all possible schedules. Thus P_ℓ must represent a very small fraction of the lower tail of the distribution. Consequently the individual p_i's, near the extreme are very small thus the probability is small that the most extreme cases will be presented in any random sample of feasible size [4].

If a sufficient amount is known about the population distribution such that it is possible to calculate approximately the probability that an observation is less than a specified value, not far from the extreme of the distribution, then the number of further observations necessary to improve on this value could be estimated. However, before an improvement is obtained, a large number of useless values may be found. These considerations imply that the

rate at which useful information obtained by random sampling is much less when estimating extremes than when estimating means.

As in any combinatorial problem the task of evaluating the distribution of the cost due to schedules is far from being easy and is time consuming. The suggestion is, therefore, to employ some kind of heuristic algorithm on the search for the least cost value. If a schedule is generated at random and reductions in the cost involved are obtained by applications of the heuristic mechanism and the process is repeated n times, then the n lowest costs generated will tend to be a random sample of extreme values from the n samples originally generated from the parent distribution. Then a Weibull distribution can be fitted to the data. The Weibull has the characteristic of being independent of the parent distribution and has as one of its own parameters the minimum value in question, [2], [3].

The approach to solve the problem presented in this study is first to develop a mathematical model to determine the relationships between schedules and facilities mix having the cost of the resources needed as the schedule's effectiveness measure. Thereafter, to generate a random starting date for each section of each course and to determine the cost of this schedule as given by the model; then by means of an heuristic procedure applied on that schedule trying to reduce the cost involved such that a local minimum cost schedule is found. Finally to use a sample

of such schedules to estimate the parameters of the Weibull distribution so that a decision can be made: whether to stop or to generate additional random schedules and to repeat the estimation of the parameters.

This thesis is organized in four sections. In Section II is developed a mathematical description of the behavior of a school which must train personnel. In such a school the courses taught can be of different lengths and each one can be composed of one or more sections, depending upon the necessity of skilled personnel. It will not deal directly with the problem of a time-table of meetings within each week. It is assumed that each type of facility is available a fixed number of hours per week and the number of trainees required in a year for a given course (or specialty) is divided into sections and they are scheduled to start at sometime during the year. It is this scheduling problem with which this thesis deals.

Section III deals with elements of the theory of extreme values and the Weibull distribution. This distribution has the feature of being bounded and describes the behavior of the extreme values taken from samples independently drawn from any parent population.

In Section IV is proposed a heuristic cost reducing procedure and a stopping decision rule. Even though the procedure is rather slow in finding a local minima and was not used in the examples discussed in Section V, it is presented here because, in the author's opinion, it has

the essential characteristics required by an algorithm to be useful for this problem.

In Section V the results of an example involving a hypothetical school are presented and the findings discussed.

In Appendix A is presented the schedule similar to that proposed in [1] that has the feature of being balanced, in the sense of facility usage by the sections of a given course throughout the complete period.

The computer program for the scheduling algorithm and a brief description of it are included in Appendix B. The program was run on the IBM 360/67 system in the "W. R. Church" Computer Center at the Naval Postgraduate School, Monterey, California.

II. THE MODEL

The major assumptions in the model are:

a. Course curricular requirements are given in the course syllabus.

b. Each unit of a specified facility is available a fixed number of hours per week.

c. The total number of students requiring instruction is nearly constant and small changes can be absorbed by varying the size of the sections.

d. A section is a physical group of students receiving a specified type of instruction.

e. No restriction is placed in the order in which the courses are taught, i.e., no course has preference to any other course. Any section of any course is equally likely to start at any time of the period.

A. FORMULATION OF THE MODEL AND NOTATION

Let $I = \{i: 1,2,...,L\}$ be the set of all different types of courses that are taught. Let N_i be defined as the number of sections of course i which must be taught during the period of one year. Let $S_i = \{s: 1,2,...,N_i\}$ be the set of sections of course i, for all $i \in I$.

Define $J = \{j: 1,2,...,M\}$ as the set of all types of facilities such as classrooms, laboratories and group of instructors. In the model $j = 1$, is reserved to designate the classrooms.

Let $K = \{k: 1,2,\ldots,T\}$ be the set of all working weeks in a specified period of time which is one year and let M_{jk} be defined as the number of units of facility j required in week k.

Let d_i be the duration in weeks of course i and A_{ij} be the total amount of facility required to teach course type i for all $i \varepsilon I$ and $j \varepsilon J$, $j \neq 1$. For example, A_{ij} is the number of lab-hours of some laboratory or number of lectures or contact-hours required by the j^{th} group of instructors. We assume that d_i and A_{ij} are predetermined and are given as exogenous parameters to the model by the syllabus of each course.

Sequencing requirements for individual topics or usage of laboratories within a determined type of course is not considered, and it is assumed that the total requirements for facility j by course i is evenly used among the weeks that comprise the duration of the course. Then it is possible to define:

$$r_{ij} = \frac{A_{ij}}{d_i}$$

as the weekly demand of facility j by one section of course i for all $i \varepsilon I$ and all $j \varepsilon J$, $j \neq 1$.

Let DA_j be the weekly availability of one unit of facility j, in hours, for all $j \varepsilon J$. DA_j is fixed for all $k \varepsilon K$.

We would like to set up a schedule such that all sections of all courses are taught during the time period of interest and the costs involved are minimized. Since under the same conditions, the schedules should be the same from one year to another, if a section has not finished its instruction at the end of the 50^{th} week it will be included at the beginning of the cycle for the time necessary to end its instruction.

Define k_{is} as the starting date of the s^{th} section of course i, for all $i \varepsilon I$ and all $s \varepsilon N_i$, and $1 \leq k_{is} \leq T$. Note that there exist as many schedules as the number of distinct combinations of the k_{is}'s; since, for a given course the sections can always be relabeled the total number of different ways in which they can be scheduled is $\frac{T^{N_i - 1}}{N_i!}$. Now for each section of course i there are $\frac{T^{N_{i'} - 1}}{N_{i'}!}$ combinations among the sections of course i', for i' = 1,2,3,...,L, i' ≠ i. Then the total number of different schedules is

$$\frac{T \exp[\sum_{i=1}^{L} N_i - 1]}{\prod_{i=1}^{L} N_i!} \qquad (1)$$

Let δ_{iks} be equal to 1 if section s of course i is in progress during the k^{th} week and 0 otherwise. Then if $k - k_{is} < 0$, the section has not started yet and $\delta_{isk} = 0$; if $0 \leq k - k_{is} < d_i$, the section is in house at week k and

$\delta_{isk} = 1$; if $k - k_{is} \geq d_i$, the section has finished its instruction and $\delta_{isk} = 0$. Note that having δ_{isk} defined in that way implies that $\delta_{isk} = \delta_{isk+1} = \ldots\ldots = \delta_{isk+d_i} = 1$ when $k_{is} = k$. This insures that once a section has started its instruction it will continue week after week until it is finished.

Summing the δ_{isk} for all $s \in S_i$ in any k produces the number of sections of course i being taught during week k:

$$\sum_{s-1}^{N_i} \delta_{isk} = \Delta_{ik}, \quad \text{for } k = 1,2,\ldots,T \tag{2}$$

Then $r_{ij} \cdot \Delta_{ik}$ represents the amount in hours per week that facility j is required by all sections of course i during week k. Summing over all $i \in I$ will produce the total number of hours of facility j needed to accommodate all courses in week k, $j \neq 1$.

If a given facility is not to be used more than it is available in any week, then we have

$$\sum_{i=1}^{L} r_{ij} \cdot \Delta_{ik} \leq M_{jk} \cdot DA_j \ , \quad \text{for all } j \in J \text{ and } k \in K, \ j \neq 1.$$

Where M_{jk} is the number of units of j which must be provided in week k.

The number of classrooms required depends on the total number of hours of meetings other than in laboratories occurring in a week. Let G be the set of indices j not corresponding to laboratories and $j \neq 1$.

Then $\sum_{j \varepsilon G} \sum_{i=1}^{L} r_{ij} \cdot \Delta_{ik}$ will represent the total number of classrooms-hours needed or the total number of meetings other than in laboratories occurring in week k and must be less than or equal to $M_{1k} \cdot DA_1$, the total availability of classrooms-hours which must be provided in that week.

Then $\sum_{j \varepsilon G} \sum_{i=1}^{L} r_{ij} \cdot \Delta_{ik} \leq M_{1k} \cdot DA_1$, for all $k \varepsilon K$.

Henceforth, M_j = maximum $(M_{jK}: k=1,2,...,T)$ represents the number of units of facility j required in the year in order that the proposed schedule, defined by k_{is}, $k = 1,2,\ldots,L$, $s = 1,\ldots,N_i$, be a feasible one.

B. COST STRUCTURE, AN OBJECTIVE FUNCTION

Since in comparing two different schedules with the same number of sections and courses, the overall operational costs in a year will be essentially the same in both schedules, the objective will be to minimize the initial cost.

Let ξ be the set of starting dates for all sections of all courses, which constitutes a schedule, thus $\xi = \{k_{is};$ all $i \varepsilon I$ and all $s \varepsilon S_i\}$. Given a particular schedule ξ the facility cost can be found from:

$$Z(\xi) = \sum_{j=1}^{M} M_j \cdot C_j \quad , \qquad (3)$$

where C_j is the initial cost of facility j for all $j \in J$. If j is a classroom or laboratory C_j includes elements such as equipment, buildings, offices, etc. In the present study the initial cost to obtain an instructor is assumed to be zero and therefore does not enter in the evaluation of a given schedule. A subjective cost could be attached to the instructors if it was desired to study their influence in the model. M_j must satisfy the following relationships:

$$M_j = \text{maximum } (M_{jk} : k = 1,2,\ldots,T) \quad \text{for all } j \in I$$

and
$$\sum_{i=1}^{L} r_{ij} \cdot \Delta_{ik} \leq M_{jk} \cdot DA_j \quad \text{for all } j \in J \text{ and } k \in K$$

$$\sum_{j \in G} \sum_{i=1}^{L} r_{ij} \cdot \Delta_{ik} \leq M_{1k} \cdot DA_1$$

$M_j \geq 1$ and integer,

Δ_{ik} defined by equation (2).

The objective is to find that schedule ξ such that $Z(\xi)$ is a minimum. It is this problem which we treat in this paper. We do not hope to find the minimum cost Z^* exactly but only to estimate its value and find a schedule with $Z(\xi) \simeq Z^*$. This estimate will be obtained by using the properties of the extreme-values and the Weibull distribution.

III. EXTREME VALUES AND THE WEIBULL DISTRIBUTION

A. A USEFUL DISTRIBUTION

In this section some properties of the extreme values and the connection to solving the minimum cost scheduling problem will be discussed. Then, using the properties of the Weibull distribution, a procedure will be presented to estimate the minimum cost.

A good presentation of the elements of the theory of extreme values is given by [5], here we only call attention to the fact that the lowest value in a random sample of size n, drawn from a parent population with cumulative distribution function $F(\cdot)$ (given by Z_{min} = minimum $(U_1, U_2, \ldots, U_n)$ where the U_i are sample values) is treated in [8] under the name Smallest Order Statistic. Weibull in [3] gives the cumulative distribution of Z_{min} in its most general form

$$F_{Z_{min}}(z) = 1 - \exp(-\phi(z)) \qquad (4)$$

where the function $\phi(z)$ must have the following characteristics[5,2]:

i) Because in the minimal cost-scheduling problem there exists a true minimum cost such that the minimum value obtained from the sample cannot be below it, this true minimum cost represents a lower bound for the extreme-value distribution. Let ζ be this true minimum value, then

$$F_{Z_{min}}(z) = 0 \qquad \text{if } z \leq \zeta,\ \zeta > 0$$

$$= 1 - \exp(-\phi(z)) \quad \text{if } z > \zeta.$$

ii) In addition $\phi(z)$ should behave as $B(Z_{min} - \zeta)$, where B is a parameter that depends upon the extreme-value spread. B must be greater than zero if $F_{min}(z)$ is a distribution function.

One distribution that satisfies the conditions stated above is the Weibull distribution (in the form given in [2]):

$$F_{Z_{min}}(z) = 1 - \exp(- [\frac{Z - \zeta}{v - \zeta}]^k) \qquad (5)$$

where

$F(z)$ = The probability that Z_{min} is less than or equal to a certain value z,

Z_{min} = The minimum cost criterion,

ζ = A constant equal to the minimum cost of the population,

v = A constant parameter indicating the value of the variable such that the probability that Z_{min} is equal to or less than v is approximately 0.63 (referred to as the characteristic smallest value in extreme-value theory),

k = A constant parameter indicating the shape of the distribution. The distribution will be positively skewed, symmetrical or negatively skewed, depending on whether k is less than, equal to, or greater than 3.259.

There is no strong theoretical support in using this distribution but empirical studies have shown the merits of its applicability [2,3]. The function has the advantage of requiring no knowledge of the parent distribution and having as one of its own parameters, the boundary value of interest.

As has been suggested before, the minimum cost obtained after applying a suitable heuristic mechanism to a random value drawn from the parent population can be regarded as an extreme-value obtained from this same population. If the same procedure is repeated n times a sample of n random and independent observations of the extreme-value, i.e. of Z_{min} will have been obtained. It is then possible to estimate the value of the lowest cost facility mix-schedule by fitting the Weibull distribution to the data.

The author is well aware that the minimum cost samples are discrete and trying to fit it to a continuous distribution will be a source of error. However, for a moderately large sample, the approximation will be close enough so as to accept the estimated lowest cost as a lower bound for any extreme-value obtained. As a general rough guide only, $n \geq 30$ is a reasonable range of values of n for applying approximations in most asymptotic results [8].

B. PARAMETER ESTIMATION OF THE WEIBULL DISTRIBUTION

Let $Z_{min_1}, Z_{min_2}, \ldots, Z_{min_n}$ be n independent and identically distributed random variables. Let $Z_{(1)} \leq$

$Z_{(2)} \leq \ldots\ldots, \leq Z_{(k)} \leq \ldots Z_{(n)}$ be the observations ordered according to their increasing size from the sample of minimum costs with cumulative distribution function as (5).

Recall that the distribution of Z_{min} should be discrete, then what is possible is that several different combinations of starting dates, i.e. schedules, will have the same cost value. Then since observations can be present with differing frequencies, define

$$Z_{(1)}, f^{(1)};\ Z_{(2)}, f^{(2)}: \ldots\ldots\ldots Z_{(m)}, f^{(m)}$$

where $Z_{(k)}$ is defined as before, $f^{(k)}$ is the corresponding frequency with which the k^{th} value occurs, and m is the number of different values that Z_{min} takes. Then, according to [7] an unbiased estimator of the cumulative distribution function of the Weibull distribution is given by:

$$\hat{F}(k) = F_Z(Z_{min} \leq z_{(k)}) = \sum_{i=1}^{m} \frac{f^{(i)}}{n+1}$$

and

$$\sum_{i=1}^{m} f^{(i)} = n$$

Having the data in the proper form and the estimate of the distribution function, the effort is now turned to the task of estimating the parameters of the Weibull distribution. Recall that this particular version of the distribution has three unknown parameters. Although there are more

accurate methods of estimating these parameters [2], we will present a method based on the double logarithm transformation of the distribution function (5) as indicated in [2,3,7]:

$$\log(\log(1-F_Z)^{-1}) = k\log(Z_{min}-\zeta) - k\log(v-\zeta) \quad (6)$$

This is the equation of a straight line on Weibull probability paper. The slope of the line is the shape parameter k. Although the parameter v is not self-evident from the graph it can be easily estimated as it will be shown later. This procedure or plotting can be repeated for different estimates of ζ, starting with $\hat{\zeta} = Z_{(1)}$ and lowering its value until the best straight line fit to the data is obtained by the least squares method.

The derivation of the regression equations for estimating the Weibull parameters will be done for grouped data. Let $Z_{(k)}$ and $f^{(k)}$ be defined as before for $k = 1,2, \ldots ,m$ and the associated cumulative distribution be:

$$\hat{F}^{(k)} = \sum_{i=1}^{m} \frac{f^{(i)}}{n+1} \quad (7)$$

For simplicity define

$$Y_k = \log(\log(1-\hat{F}^{(k)})^{-1}) \quad k=1,2,\ldots,m \quad (8)$$

$$\overline{Y} = \frac{1}{m} \sum_{k=1}^{m} Y_k$$

$$X_k = \log(Z_{(k)} - \zeta) \qquad k=1,2,\ldots,m$$

$$\overline{X} = \frac{1}{m} \cdot \sum_{k=1}^{m} X_k .$$

The regression equation as given in [9] is:

$$\hat{Y}_k = \overline{Y} + b(X_k - \overline{X}) \tag{9}$$

where $\hat{Y}_k$ is the estimate Y_k given X_k and b. The slope of the line is given by

$$b = \frac{\sum_{k=1}^{m} X_k Y_k - \frac{\sum_{k=1}^{m} X_k \sum_{k=1}^{m} Y_k}{m}}{\sum_{k=1}^{m} X_k^2 - \frac{(\sum_{k=1}^{m} X_k)^2}{m}}$$

Writing the regression line in slightly different form

$$\hat{Y}_k = bX + (\overline{Y} - b\overline{X}),$$

and comparing it with (4) we have b as the value of $\hat{k}$, the estimator of the shape parameter. In addition, since

$$k \ln(v - \zeta) = \overline{Y} - b\overline{X}$$

$\hat{v}$, the estimator of the scale parameter is obtained from

$$\hat{v} = \zeta + \exp(\frac{\overline{Y}}{\hat{k}} - \overline{X}).$$

Once the values of $\hat{k}$ and $\hat{v}$ have been obtained for a given $\hat{\zeta}$, determine the sum of squares of the differences between the values obtained by (8) using (7) and the values estimated by the regression equation, (9), i.e.

$$SQR = \sum_{k=1}^{m} (Y_k - Y_k)^2$$

Repeat this step for different values of $\hat{\zeta}$ such that $\hat{\zeta}'' < \hat{\zeta}'$, until some value of $\hat{\zeta}$, say ζ^*, yields the minimum sum of squared differences about the regression line. The associated estimators of k and v, say k* and v* are the estimated parameters of the Weibull distribution and ζ^* is the estimate of the lowest cost, i.e. the estimated value of Z_{min}.

IV. A HEURISTIC ALGORITHM

Having developed the model and examined the theory of extreme values, the remaining steps are to present a heuristic cost reducing procedure and some kind of stopping decision rule.

A. COST REDUCING PROCEDURE

As stated before, once a random schedule has been obtained, a heuristic procedure designated to find a schedule whose cost is a local minimum can be applied to it. The following heuristic procedure is suggested for moving from a random schedule ξ_i to another of lower cost, say ξ_i':

1. For each type of facility there is a maximum number of units required by a given schedule. However, this maximum number of units is not required for every week of the year. Arrange the facilities in order of increasing number of weeks during which the maximum number of units is required. The order in which they will be selected in the searching process is given by the above ordering. In the case of a tie the higher priority should be assigned to that facility whose initial cost is higher.

2. Beginning with the facility that has highest priority, to determine which courses are related to it and order them by decreasing size of the amount of facility required.

3. The search continues by, successively, picking the course which requires the largest amount of the facility, say i, and, with the exception of the s^{th} section, fix the starting dates of all sections of all courses. Change the starting date for section i, i.e., let $k_{i.s} = k$ for all $k = 1,2,\ldots,T$, and compute the cost of the modified schedule. If for some value of k, this yields a cost which is less than the previous one, retain this as a possible starting date for section s.

4. Repeat the steps 1, 2, and 3 until no further improvement is obtained.

The searching procedure possible can be shortened since it is very unlikely that a facility which is fully utilized more than fifty per cent or more can diminished by one unit. Consequently those facilities with usage factor greater than fifty per cent should not be taken into account in the search procedure, neither should those whose maximum number of units is one.

B. A STOPPING RULE

By using the above procedure we obtain a set of schedules ξ_i' whose cost $Z_{min}(\xi_i')$ can be viewed as a random sample of extreme values. These are, then, used to estimate $\zeta = Z_{min}(\xi^*)$ the minimum cost value that could be achieved. When translated into a computer program it is necessary to incorporate a rule such that after a number of trials a decision can be made whether to stop or to continue making more observations.

For instance, the following rule could be used if the expected decrease in cost by improving the scheduling is greater than or equal to the cost of one unit of the least expensive laboratory, make additional trials; otherwise stop. Formally

$$\text{if } \int_{\zeta^*}^{ZMM} (ZMM - Z_{min}(\xi))dF_{Z_{min}} \geq CI(\cdot),$$

continue searching. The number of additional trials q, is given by

$$q = \frac{1}{C_c} \int_{\zeta}^{ZMM} (ZMM - Z_{min}(\xi))dF_{Z_{min}} ;$$

if

$$\int_{\zeta^*}^{ZMM} (ZMM - Z_{min}(\xi))dF_{Z_{min}} < CI(\cdot), \text{ stop;}$$

where ZMM is the lowest value obtained at the moment of decision,

ζ^* is the estimated value of the minimum cost,

$dF_{Z_{min}}$ is the probability density function of the Weibull distribution,

$CI(\cdot)$ is the cost of one unit of the least expensive laboratory, and

C_c is the cost os one additional trial, including computer cost, analyst cost, etc.

It may still be in the designer's interest to examine additional decision parameters. Two of those are extensively explained by McRoberts [2]. Here we will give only some

illustration of the reasons why they are of interest.

These two parameters are the cumulative density function $F_{Z_{min}}$ and the value ZMM - ζ^*. The value of the shape parameter k, tends to describe the skewness of the distribution, as illustrated in figure 1. Assuming that areas A and B are equal, the probability of improvement $F_{Z_{min}}$ may represent a very small absolute potential for a small k or may represent a large band of improvement for a bigger k respectively; then it may be desirable to identify regions ZMM - ζ^* and $F_{Z_{min}}$ in the rejection range such that questionable zones may be evaluated more closely. To do so McRoberts [2] presents a plot of ZMM - ζ^* against the probability of improvement.

Figure 2 from [2] presents such a plot which is based on considering an equicost curve as a function of $F_{Z_{min}}$, CI(*) and C_c and plotted in the $F_{Z_{min}}$ - (ZMM-ζ^*) plane. By considering the equicost curve in isolation, the threshold decision line may be represented as intersecting the C curve at the value that may also be considered a threshold value of Z_{min} - ζ^*, the absolute range of improvement below which further search will not be feasible [2].

Zone A. A cost value point falling in here leads to a decision for continuous searching. This region is analogous to the statistical Type II error.

Zone B. Above threshold value, continue searching.

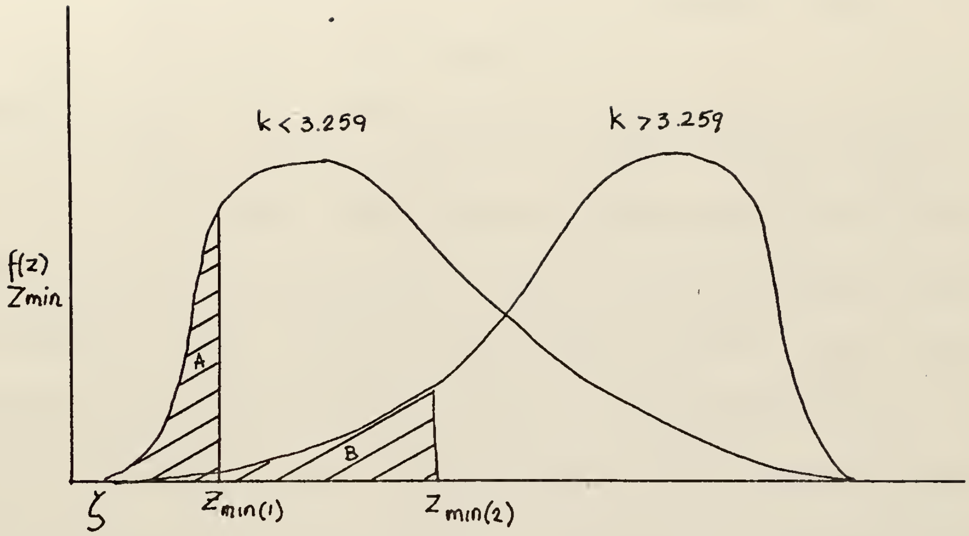

Figure 1. Effect of parameter k [2].

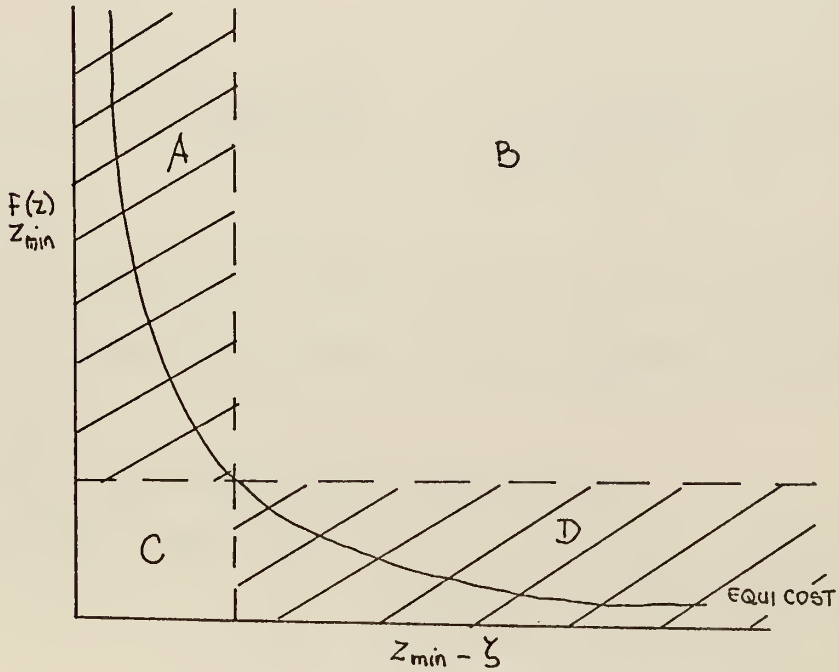

Figure 2. Decision zones for continued search [2].

Zone C. This zone lies below the threshold level to the left of the feasible absolute range. Therefore stop search.

Zone D. Here the probability of improvement may be low, but the absolute range of improvement is large. This implies continuing the search.

Other decision parameters could be the Facility Utilization Factor (UFX) and the Usage Time factor. Both have been added to the computer program. Actually they are two measures of performance. The facility utilization factor UFX(·) is defined as the ratio between actual requirements in hours of facility (·) by all courses in the period in question and the total availability in hours of facility in the same period:

$$UFX(j) = \frac{r_{ij} \cdot N_i}{T \cdot DA_j \cdot M_j} \qquad 0 \gtrsim UFX(j) \leq 1,$$

for all $j \in J$.

The usage time factor PMAX is defined as the number of weeks that the facility is used at its maximum rate divided by T = 50 weeks.

Four cases are immediately self evident:

a. UFX small, PMAX small: poor scheduling. It may be possible to eliminate one unit of that facility and consequently its cost by searching for a better schedule.

b. UFX large, PMAX small: it is very unlikely that this case can occur. If UFX is not so large and PMAX very small, it turns out to be the previous case.

c. UFX small, PMAX large: that implies that it may be possible to make a better use of that facility by incrementing the number of sections of the course or courses that make use of it. In the present conditions no improvements are obtained by further search.

d. UFX large, PMAX large: implies good employment of that resource. No further improvement may be possible.

V. SOME NUMERICAL EXAMPLES AND DISCUSSION

The search procedure presented in the previous section turns out to be fairly time consuming on the computer, at least for research purposes. Hence it is necessary to make further investigations in that area as well as to refine the stopping rule if it is possible.

In order to demonstrate the method in Sections III and IV , some hypothetical problems were run by varying the number of sections of each course, without using the search procedure. The random sample was obtained by picking the least cost schedule out of a sample of ten and repeating this step n = 100 times.

The most economical schedule found was compared against a proposed schedule similar to that suggested by Willingham [1]. A description of that schedule is given in Appendix A.

Table 2, below, shows the results of one of those examples. It was first assumed that the availability of instructors is infinite, i.e. their initial cost was zero. That problem was compared with the results of the same schedule assuming a subjective initial cost of assigning an instructor to the school, equal for all types of instructors. In either case the proposed schedule was as good as or better than the most economical of the sample, even less than the estimated minimum cost inferred from the sample values.

Other runs were made for the same problem, this time varying the seeds for the random schedules generator, obtaining the same results explained above.

Table 3 shows the results obtained by varying the number of sections of each course.

It is interesting to point out that no schedule was found with a cost less than the one proposed in Appendix A for each case. This suggest the possibility of starting with that schedule and then applying a search procedure seeking a minimum value. Of course this might not have been the case if the heuristic procedure had been used. In addition the sample size is extremely small compared to the parent population of all possible schedules as shown in (1).

This thesis and the paper by Willingham both assumed that every section of every course used facilities evenly over its duration. The method in this thesis does not require that assumption to be made and could be easily modified to reflect more accurately the actual facility usages over time for each section. If this were done it is unlikely that a "balanced schedule" of the type described in the Appendix could so easily be found. Thus the schedule proposed in the Appendix would appear less favorable compared to the method described here.

TABLE 1

Common data for a Hypothetical School

course type	1	2	3	4	5	6	7	8	9	
duration	14	10	23	9	5	18	21	21	8	
Requirements of facility										Relative Initial cost of Facilities
1										.5
2	12	15								25.0
3			5	6			5	2		175.0
4			4	3		1	7	7		150.0
5			10	2	9	2				84.0
6								5		19.0
7						2				55.0
8						7				75.0
9									11	75.0
10	23	18								1.0
11			8							1.0
12			8							1.0
13				20						1.0
14					18	4				1.0
15						5				1.0
16						12				1.0
17					7	2				1.0
18							7	7		1.0
19							4	3		1.0
20							8	8		1.0
21							3	3		1.0
22									10	1.0
23									14	1.0

Where G = (j: 2,3,4,5,6,7,8,9).

TABLE 2

COURSE TYPE	1	2	3	4	5	6	7	8	9
NUMBER OF SECTIONS	4	3	16	10	6	20	8	8	5

Problem 1

Including a subjective instructor cost (*):

PROPOSED SCHEDULE: (in Appendix A) Cost 1414.00

MOST ECONOMIC: (from sample) 1422.00

STATISTICAL INFERENCE: Estimated minimum cost 1421.98.

Weibull, shape parameter .60574.

Problem 2

Not including instructor cost:

PROPOSED SCHEDULE: (by Appendix A) 1383.00

MOST ECONOMIC: (from sample) 1383.00

STATISTICAL INFERENCE: Estimated minimum cost 1379.86.

Weibull shape parameter .66359.

(*) The differences between these two solutions was in the number of instructors required. The number of units of each type of laboratory was the same. Since the problem is to minimize the resources required, the proper way to analyse the problem is to include a cost of assigning an instructor to the school.

TABLE 3

Problem 3

COURSE TYPE	1	2	3	4	5	6	7	8	9
NUMBER OF SECTIONS	8	4	4	8	6	6	10	5	9

PROPOSED SCHEDULE: (by Appendix A) Cost 1039.00

MOST ECONOMIC: (from Sample) 1042.00

STATISTICAL INFERENCE: Estimated minimum cost 1041.93

Weibull shape parameter .70306.

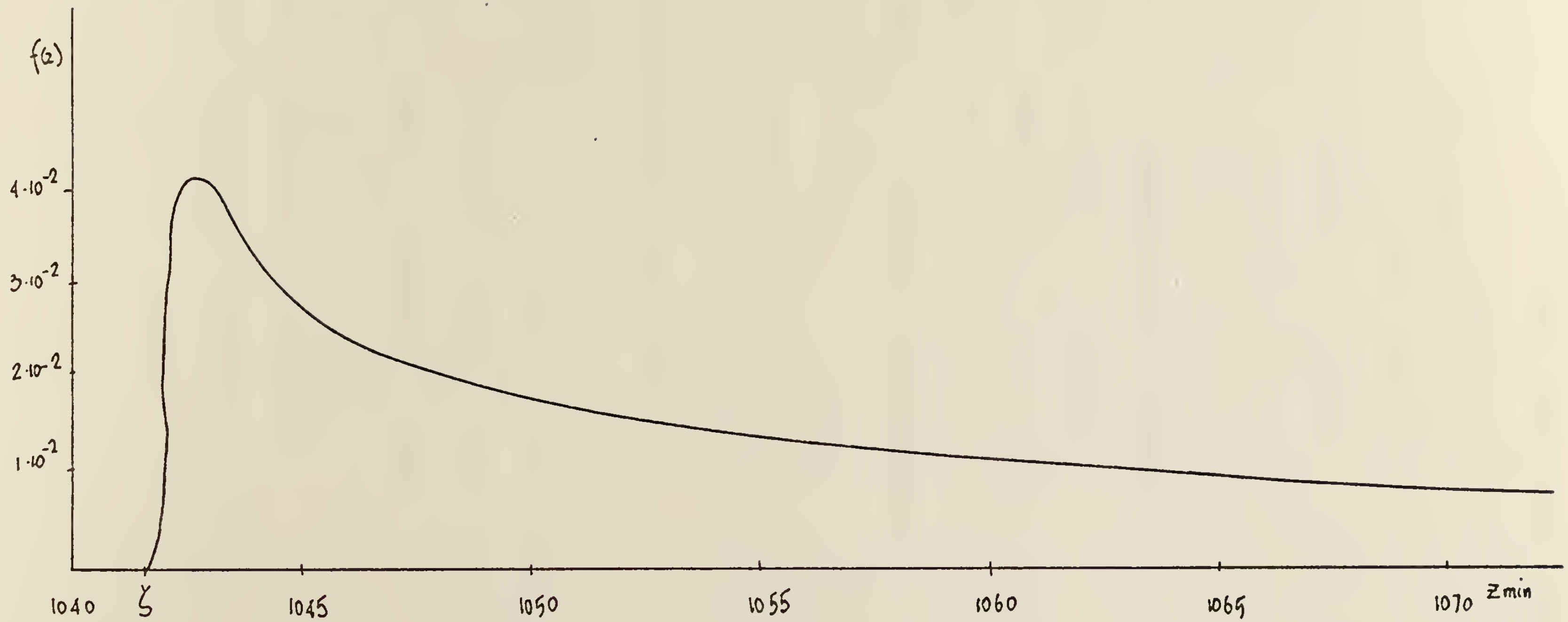

Figure 3. Probability density function obtained from sample used in problem 3 for which the result is shown in Table 3.

VI. CONCLUSION AND SUMMARY

This study addressed the problem of obtaining a schedule in a training school for the minimum cost set of facilities required, to support a given training plan, based on the quantity of students in each type of course taught. The problem was approached by making a mathematical model of the scheduling-facilities interfacing, translated later into FORTRAN IV language.

Making use of the theory of extreme-values, a sample of random minimum costs was obtained and this data was fitted to a Weibull distribution to estimate the minimum cost that it is possible to achieve.

Further investigation is required in order to come up with an efficient search procedure that assures a fast and reliable algorithm such that a minimum cost schedule is obtained by successive use of it on a random schedule drawn from the population of all feasible schedules.

The model without the heuristic procedure was tested by running several programs varying the number of sections required in a period of a year. In each case a proposed schedule (see Appendix A) was better than the most economical schedule based on a sample obtained from random schedules using the extreme-value theory, although this result would probably not hold if the courses had not been

assumed to use the facilities at a uniform rate over their duration.

From the present results it appears that a good method would be to use a schedule of the type proposed in (1) and then perhaps to apply some improving procedure to that schedule.

APPENDIX A

PROPOSING A SCHEDULE

A similar schedule has been proposed in [1] with no claim of being optimal but with the intuition that the facilities are used more or less evenly throughout the period under study.

Assume the planning period and the number of sections of each course to be taught in that period (usually a year), i.e. $T = 50$ weeks and N_i's for all $i \varepsilon I$, are known.

Define the interval between convenings for the sections of the i^{th} course by $CS_i = [T/N_i]$, largest integer. Since the right-hand-side is seldom an exact integer define $DIF = T - N_i CS_i - 1$.

Assign to sections numbered 1 the starting date $k_{is} = 1$ for all $i \varepsilon I$. Then the $(N_i - 1)$ sections will be scheduled to start their training CS_i weeks apart, except for DIF sections that will be scheduled $CS_i + 1$ weeks apart; it does not matter which of the $(N_i - 1)$ sections are selected. That will give a balanced schedule for each type of course in the sense of facility use.

APPENDIX B

THE COMPUTER PROGRAM

The program has been written in FORTRAN IV for the IBM 360/67 system at the Naval Postgraduate School. However, the program is self-contained and can be used in any other machine using FORTRAN IV programming language.

The program has been made in subroutines for easy further improvements or changes. In general it is composed of:

(a) SUBROUTINE RMDATE which generates the set of random starting dates as indicated below.

Because of the assumption that any section of any course can start its instruction at any week of the year, let the random variable K_{is} be the starting date of the s^{th} section of course i. Then the probability that K_{is} be equal to $k_{is} = k$ is given by

$$P(K_{is} = k_{is}) = \frac{1}{T} \qquad \text{for all } k \in K.$$

Then the cumulative distribution function is

$$P(K_{is} \leq k_{is}) = \frac{k}{T} \qquad \text{or}$$

$$F(k_{is}) = \frac{k}{T} \quad \text{if} \quad 1 \leq k \leq T$$

$$= 0 \quad \text{otherwise.}$$

As we can see, the distribution of K_{is} is the discrete version of the uniform distribution. Then to generate a random schedule it is only necessary to use the following modification to any uniform random number generator [0,1]:

$$K_{is} = [T \cdot RN + 0.5], \quad \text{largest integer},$$

where

K_{is} is the generated (random) starting date of the s^{th} section of course i,

T is the length in working weeks of the period that has been considered; T = 50 weeks per year,

RN is the uniform random number in the interval [0,1].

(b) SUBROUTINE COMPUT which computes, according to the model developed in Section I, the following values:

1. Number of units of each type of facility required to meet the specified program of training.

2. Number of weeks in which the above are actually required.

3. The total usage, in hours per year, of each facility by all courses.

4. The least cost schedule (ZMM) of the sample.

(c) The search procedure is incorporated to the main program by means of several subroutines:

1. SUBROUTINE SELECJ which arranges the facilities according to the procedure described in Section IV.
2. SUBROUTINE SELECI which selects a course.
3. SUBROUTINE REVAL which puts up-to-date the values obtained by SUBROUTINE COMPUT after a change on one starting date has been done.

(d) SUBROUTINE WEIBULL which computes the estimates indicated in Section III; namely:

1. ζ location parameter or minimum cost estimate.
2. k shape parameter.
3. v scale parameter.

(e) SUBROUTINE RULE which decides whether to stop or to continue searching. This part of the program makes use of the IBM SUBROUTINE QG9 (integration of a given function by the Gaussian quadrature method, nine points formula) which has been added under the name SUBROUTINE RULE. This subroutine also determines how many additional trials should be done, if any.

One of the features of the program is that one or more predetermined schedules can be tested in case there exist constraints in the scheduling period.

The program also gives a write-up of each tested schedule and the most economical schedule, including characteristic values as utilization factors, usage percentage and weekly requirements of facilities in units of facilities.

All data is fed to the program by means of DATA STATEMENTS at the beginning of it, except course requirements and the proposed schedules that are fed by means of Data Cards. The arrangement of the data cards is as follows:

1. The first card has the value of two program control parameters, ST and SNO. ST indicates the number of proposed schedules that the programmer or decision-maker wishes to test. SNO indicates whether search procedure is desired or not.
2. The next set of cards are course requirements, one for each type of facility.
3. Finally are the proposed schedule cards. Each card contains the section's starting dates of one course.

COMPUTER PROGRAM LISTING

```
  IMPLICIT INTEGER*4(D,R,T)
  INTEGER*4ST,SNO,S,SI,SCH

  DIMENSION KS(20,20),R(20,30),DA(30),CI(30),MAX(30),
 CUFX(30),KOUNT1(30),PMAXP(30),RQMNT(30),D(20),KMAX(30,50),IND(30),
 CZMIN(100),N(20),KS1(20,20),KSM(20,20),TIND(20),RT1(20)

  VARIABLE DATA
  *************

  ANY FIVE DIGITS ODD NUMBER
  DATA IR/95059/,LR/14385/, MR/19455/,ML/7544511/
  DATA L/9/,MJ/23/,TIME/50/,N/4,4,4,8,6,6,10,5,9,11*0/,MJ1/9/
  DATA D/14,10,23,9,5,18,21,21,8/,DA/9*45,21*25/
  DATA NRD/30/, NTD/10/

*****************************************************************************
**                                                                         **
**  KS(I,S)= STARTING DATA OF SECTION S OF COURSE I                        **
**  R(I,J)   WEEKLY REQUIREMENTS OF FACILITY J FOR COURSE I                **
**  N(I)     NUMBER OF SECTIONS OF COURSE I TO BE TRAIN IN ONE YEAR        **
**  D(I)     DURATION OF COURSE I                                          **
**  DELTA(I) NUMBER OF SECTIONS OF COURSE I PRESENT IN WEEK K              **
**  RI(J)    REQUIREMENTS OF FACILITY J BY ALL COURSES IN WEEK K           **
**  DA(J)    WEEKLY AVAILABILITY OF ONE UNIT OF FACILITY J                 **
**  M(J)     NUMBER OF UNITS OF FACILITY J REQUIRED AT ANY WEEK            **
**  KOUNT1   NUMBER OF PERIODS WHEN MAX(J) IS REQUIRED                     **
**  PMAX     USAGE TIME (PERCENTILE) OF MAX(J)                             **
**  MAX(J)   MAXIMUN NUMBER OF FACLITIES TYPE J REQUIRED AT ANY WEEK       **
**  UFX(J)   UTILIZATION FACTOR OF FACILITY J WHEN MAX(J) IS REQYUIRED     **
**  CI(J)    INITIAL (OR INSTALATION) COST OF FACILITY J                   **
**                                                                         **
**  M(1)     IS RESERVED FOR CLASSROOMS REQUIREMENTS CALCULATIONS          **
**  ST       INDICATES NUMBER OF PREDETERMINED SCHEDULES                   **
**  SNO      "1" COMPUTE RANDOM SCHEDULES, "0" ONLY GIVEN SCHEDULES        **
**                                                                         **
** THE ARRAY DATA OF D(I) AND RQMENT CARDS MUST HAVE SAME ORDER            **
**                                                                         **
*****************************************************************************

  FIX DATA
  *******
  DATA EPSLON/0.0/, SCALE/0.0/, CCRT/19.0/, CCOST/0.8/
  DATA CI/0.5,25.0,175.0,150.0,84.0,19.0,55.0,75.0,75.0,21*1.0/
  DATA ZM/0.0/,ZMIN/100*9999999999.9/,ZMM/999999999.9/,ZM1/0.0/
  DATA UFX/30*0.0/,IJ/0/,IND/30*0/
  DATA KOUNT1/30*0/,PMAXP/30*0.0/,RQMNT/30*0/,MAX/30*0/,SCH/0/
```

```
      READ IN PROGRAM CONTROL INDICATORS

      READ(5,1) ST, SNO
    1 FORMAT(2I5)

      READ FACILITY REQUIREMENTS FOR ALL COURSES

      DO 10 I=1,L
      READ(5,11)(R(I,J),J=1,MJ)
   11 FORMAT(1X,30I2)
   10 CONTINUE

   35 IF(ST.EQ.0) GO TO 39
      TOT=0
      ST= ST-1
      SCH= SCH+1
      NR=1
      NT= 1

      IF THERE ARE SCHEDULES TO BE TESTED READ IN THE FIRST SCHEDULE

   15 DO 16 I=1,L
      SI=N(I)
      READ(5,17)(KS(I,S), S=1,SI)
   17 FORMAT(1X,20I3)
   16 CONTINUE

      GO TO 25
   39 IF(SNO.EQ.0) STOP
      TOT=1
      LIGHT= 1
      NR= NRD
      NT= NTD
      NR1= 1

   25 DO 30 T1= NR1,NR
      IF(TOT.EQ.0) GO TO 75
      CALL RMDATE(L,N,KS,IR,LR,MR,ML,LIGHT,TIME)
      LIGHT= 0

       ALL STARTING POINTS FOR A GIVEN SET OF SECTIONS ARE NOW DETERMINED
       START COMPUTATIONS

   75 CALL COMPUT(TIME,L,N,KS,D,MJ,DA,R,KMAX,KOUNT1,RQMNT,MAX,MJ1,CI,
     CZMIN,KSM,ZMM,ZM1,T1,NR,KS1)
```

```
        REDUCING COST PROCEDURE
        ***********************

      IF(NR.EQ.1) GO TO 30
      CALL  SELECJ(TIME,MJ1,KOUNT1,CI,IND)
      MJ2= MJ1- 1

      DO 2001 JD=1,MJ2
      JK1= IND(JD)
      IF(JK1.LE.1.OR.KOUNT1(JK1).GT.24.OR.JK1.GT.MJ1) GO TO 2001
      IF(MAX(JK1).LE.1) GO TO 2001
      CALL SELECI(IND,R,L,JK1,RT1,IJ)
      IF(IJ.LE.1) GO TO 2001
      DO 2002 I=1,IJ
      LP= RT1(I)

      SI= N(LP)
      DO 2003 LS=1,SI
      DO 2004 K=1,TIME
      KS(LP,LS)= K
   80 CALL REVAL(TIME,LP,N,KS,D,KSM,R,MJ,MJ1,DA,MAX,CI,LS,KMAX,T1,
     CKOUNT1,ZMIN,ZMM,KS1)
      IF(KOUNT1(JK1).GT.24) GO TO 2001
 2004 CONTINUE
 2003 CONTINUE
 2002 CONTINUE
 2001 CONTINUE
   30 CONTINUE

      DO 2200 J=1,MJ
      PMAXP(J)= FLOAT(KOUNT1(J))/FLOAT(TIME)*100.0
      IF(MAX(J).GT.0) GO TO 2210
      UFX(J)= 9999999999.9
      GO TO 2200
 2210 UFX(J)= FLOAT(RQMNT(J))/FLOAT(MAX(J)*DA(J)*TIME)
 2200 CONTINUE

        NOW PRINT THE RESULTS
        *********************

      CALL PRINT(SND,TOT,SCH,ZMIN,ZMM,ZM1,NR,NT,MJ,MAX,UFX,PMAXP,KOUNT1,
     CL,N,KSM,KMAX,TIME)


      IF(TOT.EQ.1) GO TO 3100
      IF(ST)3150,35,35

      COMPUTE AGAIN COST AND CHARACTERISTICS OF MOST ECONOMIC SCHEDULE
```

```
3100 DO 1400 I=1,20
     DO 1500 S=1,20
     KS(I,S)= KS1(I,S)
1500 CONTINUE
1400 CONTINUE
     ST= -1
     SCH= 99
     TOT= 0
     NR= 1
     NT= 1
     GO TO 25
3150 CALL WEIBUL(NRD,ZMIN,SCALE,EPSLON,SHAPE)

     STOPPING RULE
     *************

     CALL RULE(SCALE,EPSLON,SHAPE,ZMM,CCRT,CCOST,NQ)
     IF(NQ.LT.1) STOP
     NR1= NR + 1
     NR= NR+ NQ
     GO TO 25
     END
```

```
SUBROUTINE COMPUT(TIME,L,N,KS,D,MJ,DA,R,KMAX,KOUNT1,RQMNT,MAX,MJ1,
CCI,ZMIN,KSM,ZMM,ZM1,T1,NR,KS1)
****************************************************************

PURPOSE
      THIS SUBROUTINE CALCULES, SAVE AND KEEP TRACK OF THE NUMBER OF
      UNITS OF EACH TYPE OF FACILITY REQUIRED PER WEEK AND THE MAXIMIM
      NUMBER OF UNITS PER FACILITY OVER THE PERIOD IN CONSIDERATION
      IN ORDER TO MEET THE REQUIREMENTS SET BY THE TRAINING POLICY

      EVALUES THE INSTALATION COSTS BASED ON THE NUMBER
      OF UNITS OF FACILITIES AND CLASSROOMS

      COMPARES THE COSTS OBTAINED AND SAVE THE MINIMUM VALUE, KEEPING
      RECORD OF THE SCHEDULE ASSOCIATED WITH  THIS MINIMUM COST.

ARGUMENTS
      TIME NUMBER OF WEEKS OF PLANNING PERIOD

      L    NUMBER OF DIFFERENTS TYPES OF COURSES

      N( )  NUMBERS OF SECTIONS TO BE TRAINED,OF COURSE TYPE  I.

      KS(,) STARTING DATE OF THE S-TH SECTION OF COURSE TYPE I

      D()  DURATION IN WEEKS OF COURSE I

      MJ   TOTAL NUMBER OF LABORATORIES AND BLOCKS OF INSTRUCTORS
           (FACILITIES) PLUS  ONE.  THE LATTER REPRESENTS FACILITY
           NUMBER 1 OR CLASSROOMS

      DA() WEEKLY AVAILABILITY OF ONE UNIT OF FACILITY J

      R(I,J) THE REQUIREMENTS OF COURSE I FOR FACILITY J
           PER WEEK.

      KMAX() REPRESENTS THE NUMBER OF UNITS OF EACH TYPE OF FACILITY
           REQUIRED IN A WEEK TO MEET THE  WORK LOAD

      KOUNT1 INDICATES NUMBER OF WEEKS WHEN THE MAXIMUM NUMBER OF UNITS
           OF FACILITY J IS USED

      RQMNT()  IS THE TOTAL AMOUNT OF HOURS  THAT  THE FACILITY IS
           REQUIRED  IN THE PLANNING PERIOD OF ONE YEAR

      MAX() MAXIMUM NUMBER OF UNITS OF FACILITY J NEEDED TO MEET THE
           TRAINING REQUIREMENTS
```

```
            MJ1  ONE PLUS NUMBER OF DIFFERENT TYPES OF LABORATORIES
                 (LAB. TYPE FACILITIES) REQUIRED

            CI(J) INSTALATION COST OF ONE UNIT OF FACILITY J

            ZMIN SAMPLE OF A LOCAL MINIMUM  COST SCHEDULE OBTAINED  FROM
                 THE SEARCH PROCEDURE

            KSM(,)  STARTING DATES OF THE MOST ECONOMICAL SCHEDULE FOUND

            ZMM  COST OF FACILITIES FOR THE ABOVE SCHEDULE

            ZM1 FACILITIES COST OF ANY PROPOSED SCHEDULE

            T1 SAMPLE NUMBER

            KS1(,) STARTING DATES OF ANY PROPOSED SCHEDULE

      IMPLICIT INTEGER*4(D,R,T)
      INTEGER*4ST,SNO,S,SI,SCH
      DIMENSION N(20),KS(20,20),D(20),DELTA(20),RJ(30),R(20,30),MAX(30),
     CRQMNT(30),M(30),KMAX(30,50),KOUNT1(30),DA(30),CI(30),ZMIN(100),
     CKSM(20,20),KMA(30,50),KOUNT(30),RQMAD(30),KS1(20,20)

      DO 90 J=1,MJ
      MAX(J)= 0
      KOUNT(J)= 0
      RJ(J)= 0
      M(J)= 0
      RQMAD(J)= 0
   90 CONTINUE

      DO 100 K=1,TIME
      DO 300 I=1,L
      DELTA(I)= 0
      SI=N(I)

      IS SECTION S OF COURSE I AT HOME DURING WEEK K?

      DO 400 S=1,SI
      IF(K- KS(I,S)) 450,500,550
  450 IF(KS(I,S)+ D(I)- K- TIME- 1)400,500,500
  550 IF(KS(I,S)+ D(I)- K -1) 400,500,500
  500 DELTA(I)= DELTA(I)+ 1
  400 CONTINUE
```

```
      COMPUTE THE TOTAL AMOUNT OF FACILITY J REQUIRED IN WEEK K

  300 CONTINUE
      DO 200 J=2,MJ
      DO 350 I=1,L
      RJ(J)= R(I,J)*DELTA(I)+ RJ(J)
  350 CONTINUE
      IF(J.LE.MJ1) GO TO 200
      RJ(1)= RJ(1) +RJ(J)
  200 CONTINUE

      HERE WE COMPUTE THE NUMBER OF FACILITIES J REQUIRED TO MEET THE DEMAND

      DO 650 J=1,MJ
      J1= 0
  600 J1= J1 +1
      IF(RJ(J).GT.J1*DA(J)) GO TO 600
      M(J)= J1
      IF(RJ(J).EQ.0)  M(J)= 0
      RQMAD(J)= RQMAD(J) + RJ(J)
      IF(M(J).LT.MAX(J)) GO TO 199
      IF(M(J).EQ.MAX(J)) GO TO 198
      MAX(J)= M(J)
      KOUNT(J)= 1
      GO TO 199
  198 KOUNT(J)= KOUNT(J)+ 1
  199 KMA(J,K)= M(J)
      RJ(J)= 0
      M(J)= 0
  650 CONTINUE
  100 CONTINUE

      ZM= 0.0
      DO 1100 J=1,MJ
      ZM= ZM + MAX(J)*CI(J)
 1100 CONTINUE

      HERE WE SAVE THE VALUE OF THE MINIMUN COST OF THE RUN

      ZM1= ZM
      IF(NR.EQ.1) GO TO 1150
      IF(ZMIN(T1).LE.ZM) RETURN
      ZMIN(T1)= ZM
 1150 IF(ZMM.LT.ZM) GO TO 1175

      THIS IS THE MOST ECONOMICAL SCHEDULE FOUND

      ZMM= ZM
```

```
      DO 1180 I=1,L
      SI=N(I)
      DO 1190 S=1,SI
      KS1(I,S)= KS(I,S)
 1190 CONTINUE
 1180 CONTINUE

      SAVE THE SCHEDULE WITH THE LEAST COST  ACCORDING TO THE SEARCH PROCEDURE

 1175 DO 1200 I=1,L
      SI= N(I)
      DO 1300 S=1,SI
      KSM(I,S)= KS(I,S)
 1300 CONTINUE

      TI1= SI
      DO 1350 S=1,SI
      TI1= TI1- 1
      IF(TI1.LT.1) GO TO 1350
      DO 1375 T2= 1,TI1
      T3= T2+ 1
      IF(KSM(I,T2).LE.KSM(I,T3)) GO TO 1375
      KTEMP= KSM(I,T2)
      KSM(I,T2)= KSM(I,T3)
      KSM(I,T3)= KTEMP
 1375 CONTINUE
 1350 CONTINUE
 1200 CONTINUE

      DO 1400 J=1,MJ
      KOUNT1(J)= KOUNT(J)
      RQMNT(J)= RQMAD(J)
      DO 1450 K=1,TIME
      KMAX(J,K)= KMA(J,K)
 1450 CONTINUE
 1400 CONTINUE

      RETURN
      END
```

```
SUBROUTINE WEIBUL(NRD,ZMIN,SCALE,EPSLON,SHAPE)
*********************************************

PURPOSE
      THIS SUBROUTINE ESTIMATES THE PARAMETERS OF A WEIBULL DISTRI-
      BUTION BASED ON THE MINIMUM SUM OF THE SQUARES ABOUT THE
      REGRETION LINE.

      LOG(Z- EPSLON)- LOG(SCALE- EPSLON)=
                     =(1/SHAPE)*LOG(-LOG(1.0-F(Z)))
      WHERE

         EPSLON: A CONSTAN EQUAL TO THE LOWEST VALUE EXTREME VALUE

         SCALE   A COSTANT PARAMETER INDICATING THE VALUE OF THE
                 OR LOCATION PARAMETER OF THE DISTRIBUTION
                 VARIABLE SUCH THAT THE PROBABILITY THAT THE EXTREME
                 VALUE IS EQUAL TO OR LESS THAN SCALE IS APROXIMATELY
                 0.63, REFERRED TO AS THE CHARASTERISTIC SMALLEST
                 VALUE IN EXTREME-VALUE THEORY
                 (SCALE-EPSLON) IS THE IS THE DISTRIBUTION'S
                 SCALE PARAMETER

         SHAPE   A CONSTANT PARAMETER INDICATING THE SHAPE OF THE
                 DISTRIBUTION
ARGUMENTS
      NRD  NUMBER OF OBSERVATIONS

      ZMIN SAMPLE OF A LOCAL MINIMUM  COST SCHEDULE OBTAINED  FROM
           THE SEARCH PROCEDURE

IMPLICIT INTEGER*4(D,R,T)
INTEGER*4SIZE,FLAG
INTEGER*4ST,SNO,S,SI,SCH
DIMENSION FZ(100), C(100), A(100), CESTM(100),TCOUNT(100)
DIMENSION ZMIN(100)
DATA ABAR/0.0/,CBAR/0.0/,AC/0.0/,A2/0.0/,C2/0.0/,SUM/0.0/
DATA JUMP/0/, FLAG/0/, F2/0.0/,TCOUNT/100*0/
  ORDERING THE COST SAMPLES ACCORDING TO THEIR INCREASING SIZE
SIZE= NRD
N1= NRD
DO 3000 T=1,SIZE
N1= N1- 1
```

```
      DO 3200 T2=1,N1
      T3= T2+ 1
      IF(ZMIN(T2).LE.ZMIN(T3)) GO TO 3200
      ZTEMP= ZMIN(T2)
      ZMIN(T2)= ZMIN(T3)
      ZMIN(T3)= ZTEMP
3200  CONTINUE
3000  CONTINUE

       GROUPING THE DATA AND OBTAINING ITS FREQUENCY

      TN= 1
3500  IF(TN.GT.SIZE) GO TO  3675
      I1= TN
      DO 3600 T1= I1,SIZE
      TMIN=IFIX(ZMIN(T1)*100.0)
      TMIN2= IFIX(ZMIN(TN)*100.0)
      IF(TMIN- TMIN2) 3625,3650,3600
3625  WRITE(6,3626)
3626  FORMAT(1X,//,20X,'ERROR IN LOOP "3600"',//)
      RETURN
3650  TCOUNT(TN)= TCOUNT(TN)+ 1
3600  CONTINUE
      TN= TN+ TCOUNT(TN)
      GO TO 3500
3675  TC= 0
      DO 3700 T=1,SIZE
      IF(TCOUNT(T))3725,3700,3775
3725  WRITE(6,3726)
3726  FORMAT(1X,//,20X,'ERROR IN LOOP "3700"',//)
      RETURN
3775  TC= TC+ 1
      TCOUNT(TC)= TCOUNT(T)
      ZMIN(TC)= ZMIN(T)
3700  CONTINUE

       ESTIMATING THE VALUES OF THE  CUMULATIVE DISTRIBUTION FUNCTION

      F1= 1.0/FLOAT(SIZE+1)
      DO 3300 T2= 1,TC
      F2= F2+ TCOUNT(T2)*F1
      FZ(T2)= F2
      C(T2)=ALOG(-ALOG(1.0- FZ(T2)))
3300  CONTINUE

      EPSLON= ZMIN(1)
      ZRANGE= ZMIN(TC)- ZMIN(1)
```

```
      STEP= ZRANGE/ 100.0
      ALT= STEP/1000
 4300 EPSLON=EPSLON- STEP

      DO 4400 T=1,TC
      A(T)= ALOG(ZMIN(T)-EPSLON)
 4400 CONTINUE

      REGRETION LINE ESTIMATION

      DO 4500 T3=1,TC
      ABAR= ABAR +A(T3)
      CBAR= CBAR +C(T3)
      AC= AC +C(T3)*A(T3)
      A2= A2 +A(T3)*A(T3)
      C2= C2 +C(T3)*C(T3)
 4500 CONTINUE

      SIZE= TC
      SLOPE=(AC- ABAR*CBAR/SIZE)/(A2- ABAR*ABAR/SIZE)
      AVAR= (A2- ABAR*ABAR/SIZE)/(SIZE- 1)
      CVAR= (C2- CBAR*CBAR/SIZE)/(SIZE- 1)
      B= (ABAR- CBAR/SLOPE)/ SIZE

      OBTAINING THE SUM OF THE SQUARE DIFFERENCES

      DO 4600 T4=1,TC
      CESTM(T4)= CBAR/SIZE +SLOPE*(A(T4)- ABAR/SIZE)
      SUM= SUM +(C(T4)- CESTM(T4))*(C(T4)- CESTM(T4))
 4600 CONTINUE

      IF THE ESTIMATE OF EPSLON DOES NOT YIELD THE MINIMUM SUM OF THE SQUARE
      DIFFERENCES,TRY AGAIN WITH ANOTHER EPSLON UNTIL OBTAIN SO

      IF(JUMP.EQ.1) GO TO 4700
      SQRMIN= SUM
      SHAPE= SLOPE
      SCALE= EXP(B)+ EPSLON
      JUMP= 1

 4700 IF(SUM.LT.SQRMIN) GO TO 4800
      IF(FLAG.EQ.1) GO TO 4900
      ABAR= 0.0
      CBAR= 0.0
      AC= 0.0
      A2= 0.0
      C2= 0.0
      SUM= 0.0
```

```
      FLAG=1
      GO TO 4300

4800  SQRMIN= SUM
      SHAPE= SLOPE
      SCALE= EXP(B)+ EPSLON
      ABAR= 0.0
      CBAR= 0.0
      AC= 0.0
      A2= 0.0
      C2= 0.0
      SUM= 0.0
      GO TO 4300

4900  IF(STEP.LT.ALT) GO TO 9800
      EPSLON= EPSLON+ 2*STEP
      STEP= STEP/10.0
      ABAR=0.0
      CBAR= 0.0
      AC= 0.0
      A2= 0.0
      C2= 0.0
      SUM= 0.0
      FLAG= 0
      JUMP= 0
      GO TO 4300

9800  NR= NRD
      NT= NTD
      WRITE(6,3400)NR,TC,(ZMIN(J1),J1=1,TC)
3400  FORMAT('1',//,20X,'SAMPLE SIZE',I5,6X,'REDUCED SAMPLE SIZE WEIBULL
     C',I3,/,20X,'*******************************************************',
     C///,20X,'ORDERED MINIMUN COSTS',/,(' ',20X,5F15.3))

      WRITE(6,3450)(TCOUNT(J),J=1,TC)
3450  FORMAT('0',19X,'SAMPLE FREQUENCIES',/,(' ',16X,5I15))

      WRITE(6,9900)EPSLON,SHAPE,SCALE
9900  FORMAT(' ',///,20X,'PARAMETERS WEIBULL DISTRIBUTION',/,20X,'*****
     C*************************',//,20X,'MINIMUN VALUE, ESTIMATE:',F15.5
     C,//,20X,'SHAPE PARAMETER:',8X,F15.5,//,20X,'SCALE PARAMETER:',8X,F
     C15.5)
      RETURN
      END
```

```
      SUBROUTINE RMDATE(L,N,KS,IR,LR,MR,ML,LIGHT,TIME)
      ***************************************************

      PURPOSE
             THIS SUBROUTINE GENERATES RANDOM STARTING DATES FOR ALL
                  SECTIONS OF ALL COURSES.
      ARGUMENTS
             L    NJMBER OF DIFFERENT TYPES OF COURSES

             N(I) NUMBER OF SECTIONS TO BE TRAINED OF COURSE I DURING
                  THE PERIOD "TIME".

             KS(,) STARTING DATE OF THE S-TH SECTION OF COURSE TYPE I

             IR   FIVE DIGITS ODD NUMBER
             LR     "    "     "    "
             MR     "    "     "    "
             ML     "    "     "    "

             LIGHT SENSOR=1 AT THE BEGINNING OF THE PROGRAM
                          0 THERE AFTER

             TIME NUMBER OF WEEKS OF PLANNING PERIOD

      INTEGER*4S,SI,TIME
      DIMENSION N(20), KS(20,20),NI(128)
        DATA IK/65539/, MM/33554433/, MN/36243609/
      IF(LIGHT.EQ.0)GO TO 80
      DO 40 IG=1,128
      IR=IR*IK
      NI(IG)= IR
   40 CONTINUE
   80 DO 70 I=1,L
      SI=N(I)
      DO 60 S=1,SI
      LR= LR*ML
      MR=MR*MM
      JR= (IABS(LR)/16777216) + 1
      GNR= 0.5 + FLOAT(NI(JR)+LR+MR)*2.328306 E-10
      IK= IK*MN
      NI(JR)= IK
      KS(I,S)= IFIX(TIME*GNR + 0.5)
      IF(KS(I,S).LT.1) KS(I,S)=1
      IF(KS(I,S).GT.50) KS(I,S)= 50
   60 CONTINUE
   70 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PRINT(SNO,TOT,SCH,ZMIN,ZMM,ZM1,NR,NT,MJ,MAX,UFX,PMAXP,
     CKOUNT1,L,N,KSM,KMAX,TIME)
      ***************************************************************


      IMPLICIT INTEGER*4(D,R,T)
      INTEGER*4ST,SNO,S,SI,SCH
      DIMENSION ZMIN(100),MAX(30),UFX(30),PMAXP(30),KOUNT1(30),N(20),
     CKSM(20,20),KMAX(30,50)

      IF(SNO.EQ.1.AND.TOT.EQ.1) GO TO 8900
      IF(SCH.NE.99) GO TO 8700

      WRITE(6,8600) ZMM
 8600 FORMAT('1',//,20X,' THIS IS THE MOST ECONOMICAL SCHEDULE FOUND',/,
     C 20X,'**********************************************',///,20X,'FACILITI
     CES INSTALATION COST=',F15.3,/)
      GO TO 9150

 8700 WRITE(6,8800)SCH, ZM1
 8800 FORMAT('1',//,20X,'PROPOSED SCHEDULE NUMBER',I3,/,20X,'***********
     C************',///,20X,'FACILITIES INSTALATION COST=',F15.3,/)
      GO TO 9150

 8900 SNO= 0
      WRITE(6,9000)NR,NT,(ZMIN(J1),J1=1,NR)
 9000 FORMAT('1',//,20X,'SAMPLE SIZE',I5,6X,'NUMBER OF TRIALS PER SAMPLE
     C',I3,/,20X,'***************************************************',
     C///,20X,'MINIMUN COSTS SAMPLE',/,('0',13X,5F15.3))
      RETURN

 9150 WRITE(6,9400)(J,MAX(J),J=1,MJ)
 9400 FORMAT(1X,///,20X,'FACILITIES REQUIRED',/,(' ',20X,5(I2,')',I5,7X
     ")))

      WRITE(6,9500)(J,UFX(J),J=1,MJ)
 9500 FORMAT(1X,//,20X,'FACILITIES UTILIZATION FACTOR UFX(J)',/,(' ',20X
     C,5(I2,')',F5.3,7X)))

      WRITE(6,9600)(J,PMAXP(J),J=1,MJ)
 9600 FORMAT(1X,//,20X,'USAGE TIME (PERCENTILE) OF MAX(J)',/,(' ',20X,5(
     CI2,')',F7.3,5X)))

      WRITE(6,9700)(J,KOUNT1(J),J=1,MJ)
 9700 FORMAT(1X,//,20X,'NUMBER OF WEEKS WHEN MAX(J) IS REQUIRED',/,(' ',
     C20X,5(I2,')',I5,7X)))
```

```
      DO 1200 I=1,L
      SI=N(I)
      TI1= SI
      DO 1350 S=1,SI
      TI1= TI1- 1
      IF(TI1.LT.1) GO TO 1350
      DO 1375 T2= 1,TI1
      T3= T2+ 1
      IF(KSM(I,T2).LE.KSM(I,T3)) GO TO 1375
      KTEMP= KSM(I,T2)
      KSM(I,T2)= KSM(I,T3)
      KSM(I,T3)= KTEMP
 1375 CONTINUE
 1350 CONTINUE
 1200 CONTINUE

      WRITE(6,9706)
 9706 FORMAT(1X,//,20X,'SECTIONS STARTING DATES SCHEDULED')
      DO 9702 I=1,L
      SI= N(I)
      WRITE(6,9701)(KSM(I,S),S=1,SI)
 9701 FORMAT(20X,20I4)
 9702 CONTINUE

      WRITE(6,9705)(J,J=1,MJ)
 9705 FORMAT(1X,//,20X,'WEEKLY LOAD',/,25X,30I3)
      WRITE(6,9707)
 9707 FORMAT('0')
      DO 9703 K=1,TIME
      WRITE(6,9704)K,(KMAX(J,K),J=1,MJ)
 9704 FORMAT(20X,I3,2X,30I3)
 9703 CONTINUE

      RETURN
      END
```

```
      SUBROUTINE SELECJ(TIME,MJ1,KOUNT1,CI,IND)
      *****************************************

      PURPOSE
             THIS SUBROUTINE SELECTS THAT FACILITY (OF LAB TYPE) WITH
             SMALLEST USAGE FACTOR OF THE MAXIMUM REQUIRED INTHE CONSIDERED
             PERIOD OF TIME. I.E. THAT FACILITY WITH  KOUNT1(.) INDEX
             IS SMALLER.

      ARGUMENTS
             TIME NUMBER OF WEEKS OF PLANNING PERIOD

             MJ1  ONE PLUS NUMBER OF DIFFERENT TYPES OF LABORATORIES
                  (LAB. TYPE FACILITIES) REQUIRED

             KOUNT1 INDICATES NUMBER OF WEEKS WHEN THE MAXIMUM NUMBER OF UNITS
                  OF FACILITY J IS USED

             CI(J) INSTALATION COST OF ONE UNIT OF FACILITY J

             IND(N) INDEX WHICH VALUE IS THE NUMBER DESIGNATION OF A FACILITY
                  AND ITS ARGUMENT N INDICATES THE ORDERING OF THE FACILITIES
                  BY SMALLEST USAGE TIME AND MORE EXPENSIVE INSTALATION COST

      IMPLICIT INTEGER*4(D,R,T)
      INTEGER*4ST,SNO,S,SI,SCH
      DIMENSION KOUNT1(30),CI(30),IND(30)
      JK= 0
      NO= 1
      DO 1000 K=1, TIME
      DO 1001 J=2,MJ1
      IF(KOUNT1(J).EQ.K) GO TO 1003
      GO TO 1001
 1003 JK= JK+ 1
      IND(JK)= J
 1001 CONTINUE
      IF(JK.LE.NO) GO TO 1006
      N1= JK
      DO 1004 T= NO,JK
      N1= N1- 1
      IF(N1.LT.NO) GO TO 1004
      DO 1005 N2= NO,N1
      N3= N2+ 1
      N4= IND(N2)
      N5= IND(N3)
      IF(CI(N4).LE.CI(N5)) GO TO 1005
```

```
      IND(N2)= IND(N3)
      IND(N3)= NTEMP
 1005 CONTINUE
 1004 CONTINUE
 1006 NO= JK+ 1
 1000 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SELECI(IND,R,L,JK1,RT1,IJ)
      *************************************

      PURPOSE
             THIS SUBROUTINE SEARCH FOR A GIVEN FACILITY J, WHOSE COURSES ARE
             RELATED TO IT.
             GIVES WHICH COURSE REQUIRES MORE AMOUNT OF THE FACILITY.
             THEN THE NEXT ONE AND SO ON.

      ARGUMENTS
             IND(.) IDENTIFICATION NUMBER OF THE GIVEN FACILITY

             R(I,J) THE REQUIREMENTS OF COURSE I FOR FACILITY J
                  PER WEEK.

             L    NJMBER OF DIFETENTS TUPES OF COURSES

             JK1  NUMBER SELECTED BY SUBROUTINE SELEJ.
                  INDICATES FACILITY WITH LESS USAGE TIME

             RT1() INDEX REPRESENTING COURSE IDENTIFICATION NUMBER WHICH
                  REQUIRES MORE AMOUNT (IN HORS-WEEK) OF FACILITY JK1

             IJ   NUMBER OF COURSES RELATED WITH FACILITY JK1


      IMPLICIT INTEGER*4(D,R,T)
      INTEGER*4ST,SNO,S,SI,SCH
      DIMENSION IND(30),ID(20),RT(20),RT1(20),R(20,30)

      JD= JK1
      IJ=0
      DO 1006 I=1,L
      IF(R(I,JD))1006,1006,1007
 1007 IJ= IJ+ 1
      ID(IJ)= I
 1006 CONTINUE
      DO 1008 I=1,IJ
      II=ID(I)
      RT(I)= R(II,JD)
      RT1(I)= II
 1008 CONTINUE

      IJ1= IJ
      DO 1009 T=1,IJ
      IJ1= IJ1 -1
      IF(IJ1.LT.1) GO TO 1009
```

```
      DO 1010 T2= 1,IJ1
      T3= T2 +1
      IF(RT(T2).GE.RT(T3)) GO TO  1010
      RTEMP= RT(T2)
      RTEMP1= RT1(T2)
      RT(T2)= RT(T3)
      RT1(T2)= RT1(T3)
      RT(T3)= RTEMP
      RT1(T3)= RTEMP1
 1010 CONTINUE
 1009 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE REVAL(TIME,LP,N,KS,D,KSM,R,MJ,MJ1,DA,MAX,CI,LS,KMAX,T1,
     CKOUNT1ZMIN,ZMM,KS1)
      ******************************************************************

      PURPOSE
             THIS SUBROUTINE EVALUES THE CHANGES IN COST DUE TO THE
             SEACH PROCEDURE

      ATGUMENTS
             TIME NUMBER OF WEEKS OF PLANNING PERIOD

             LP   IDENTIFICATION NUMBER OF A COURSE SELECTED BY SUBROUTINE SELCJ

             N( )  NUMBERS OF SECTIONS TO BE TRAINED,OF COURSE TYPE  I.

             KS(,) STARTING DATE OF THE S-TH SECTION OF COURSE TYPE I

             D()  DURATION IN WEEKS OF COURSE I

             KSM(,)  STARTING DATES OF THE MOST ECONOMICAL SCHEDULE FOUND

             R(I,J) THE REQUIREMENTS OF COURSE I FOR FACILITY J
                  PER WEEK.

             MJ   TOTAL NUMBER OF LABORATORIES AND BLOCKS OF INSTRUCTORS
                  (FACILITIES) PLUS  ONE.  THE LATTER REPRESENTS FACILITY
                  NUMBER 1 OR CLASSROOMS

             MJ1  ONE PLUS NUMBER OF DIFFERENT TYPES OF LABORATORIES
                  (LAB. TYPE FACILITIES) REQUIRED

             DA() WEEKLY AVAILABILITY OF ONE UNIT OF FACILITY J

             MAX() MAXIMUM NUMBER OF UNITS OF FACILITY J NEEDED TO MEET THE
                  TRAINING REQUIREMENTS

             CI(J) INSTALATION COST OF ONE UNIT OF FACILITY J

             LS SECTION SELECTED IN THE  SEARCH PROCEDURE

             KMAX() REPRESENTS THE NUMBER OF UNITS OF EACH TYPE OF FACILITY
                  REQUIRED IN A WEEK TO MEET THE  WORK LOAD

             T1 SAMPLE NUMBER

             KOUNT1 INDICATES NUMBER OF WEEKS WHEN THE MAXIMUM NUMBER OF UNITS
                  OF FACILITY J IS USED
```

```
ZMIN SAMPLE OF A LOCAL MINIMUM  COST SCHEDULE OBTAINED  FROM
     THE SEARCH PROCEDURE

ZMM, COST VALUE OF THE MOST ECONOMICAL SCHEDULE FOUND AT THE MOMENT
     OF THE DECISION

KS1(,) STARTING DATES OF ANY PROPOSED SCHEDULE

      IMPLICIT INTEGER*4(D,R,T)
      INTEGER*4ST,SNO,S,SI,SCH
      DIMENSION N(20),KS(20,20),D(20),KSM(20,20),RJ(30),R(20,30),DA(30),
     CM(30),MAX(30),RJMAX(30),KMAX(30,50),CI(30),KOUNT1(30),ZMIN(100),
     CKS1(20,20)
      DATA DELT1/0/,DELT2/0/,RTEST/0/,RJ/30*0/,M/30*0/

      DO 100 K=1,TIME
      SI=N(LP)
      DO 400 S=1,SI
      IF(K- KS(LP,S))450,500,550
  450 IF(KS(LP,S)+D(LP)-K-TIME-1) 400,500,500
  550 IF(KS(LP,S)+D(LP)-K-1) 400,500,500
  500 DELT1= DELT1+ 1
  400 CONTINUE

      DO 401 S=1,SI
      IF(K-KSM(LP,S)) 451,501,551
  451 IF(KSM(LP,S)+ D(LP) -K-TIME- 1) 401, 501,501
  551 IF(KSM(LP,S)+D(LP)-K-1) 401,501,501
  501 DELT2= DELT2+ 1
  401 CONTINUE

      IF(DELT2.LE.DELT1) GO TO 390
      DO 200 J=2,MJ
      RJ(J)= R(LP,J)*DELT1 + RJ(J)
      IF(J.LE.MJ1) GO TO 200
      RJ(1)= RJ(1)+ RJ(J)
  200 CONTINUE

      DO 650 J=1,MJ
      J1= 0
  600 J1= J1+ 1
      IF(RJ(J).GT.J1*DA(J)) GO TO 600
      M(J)= J1
      IF(RJ(J).EQ.0) M(J)= 0
      IF(M(J)- MAX(J)) 370,390,390
  370 IF(M(J)- KMAX(J,K)) 380,390,390
  380 KOUNT1(J)= KOUNT1(J)- 1
```

```
      IF(KOUNT1(J)) 381,381,383
  381 KOUNT1(J)= 50
      MAX(J)= MAX(J)- 1
  383 KMAX(J,K)= M(J)
      RJ(J)= 0
      M(J)= 0
      RTEST= 0
  650 CONTINUE
  390 DELT1= 0
      DELT2= 0
  100 CONTINUE

      IF(RTEST.EQ.0) RETURN
      ZM= 0.0
      DO 1100 J=1,MJ1
      ZM= ZM+ MAX(J)* CI(J)
 1100 CONTINUE

      IF(ZMM.GT.ZM) KS1(LP,LS)= KS(LP,LS)
      IF(ZMIN(T1).LE.ZM) RETURN
      ZMIN(T1)= ZM
      KSM(LP,LS)= KS(LP,LS)
      RETURN
      END
```

```
      SUBROUTINE RULE(SCALE,EPSLON,SHAPE,ZMM,CCRT,CCOST,NQ)
      ***********************************************************

      PURPOSE
            THIS SUBROUTINE DETERMINES WHETHER TO STOP SEARCH OR TO
            CONTINJE IN WHICH CASE ALSO DETERMINES THE NUMBER OF ADDITINAL
            TRIALS.

      ARGUMENTS
            SCALE, PARAMETER OF THE WEIBULL DISTRIBUTION

            EPSLON, ESTIMATED MINIMUM COST

            SHAPE, PARAMETER OF THE WEIBULL DISTRIBUTION

            ZMM, COST VALUE OF THE MOST ECONOMICAL SCHEDULE FOUND AT THE MOMENT
                 OF THE DECISION

            CCRT, COST OF ONE UNIT OF THE LEAST EXPENSIVE LABORATORY

            CCOST, COST OF ONE ADDITIONAL TRIAL.

            NQ NUMBER OF ADDITIONAL TRIALS THAT CAN BE DONE

      A=.5*(ZMM+EPSLON)
      B=ZMM-EPSLON
      C=.4840801*B
      Y=.04063719*(PDF(SCALE,EPSLON,SHAPE,ZMM,A+C)+PDF(SCALE,EPSLON,
     CSHAPE,A-C))
      C=.4180156*B
      Y=Y+ .09032408*(PDF(SCALE,EPSLON,SHAPE,ZMM,A+C)+PDF(SCALE,EPSLON,
     CSHAPE,ZMM,A-C))
      C=.3066857*B
      Y=Y+ .1303053*(PDF(SCALE,EPSLON,SHAPE,ZMM,A+C)+PDF(SCALE,EPSLON,
     CSHAPE,ZMM,A-C))
      C=.1621267*B
      Y=Y+ .1561735*(PDF(SCALE,EPSLON,SHAPE,A+C)+PDF(SCALE,EPSLON,
     CSHAPE,ZMM,A-C))
      Y= B*(Y+ .1651197*PDF(SCALE,EPSLON,SHAPE,ZMM,A))
      IF(Y.LT.CCRT) GO TO 3200
      NQ= IFIX(Y/CCOST)
      RETURN
 3200 WRITE(6,3300)
 3300 FORMAT('0',20X,'NO MORE TRIALS ARE NECESARY,  STOP SEARCH')
      NQ=0
      RETURN
      END
```

```
      FUNCTION PDF(SCALE,EPSLON,SHAPE,ZMM,Z)
      **************************************

      CONST1= (SCALE-EPSLON)**SHAPE
      CONST2= CONST1*SHAPE
      PDF=(ZMM - Z)*CONST2*((Z - EPSLON)**(SHAPE-1))*EXP(-((Z-EPSLON)**S
     CHAPE)/CONST2)
      RETURN
      END
```

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13. ABSTRACT

The problem of determining the quantity of classrooms, laboratories and instructors to train sections of students attending numerous distinct courses in a school such as the Fleet Ballistic Missile School is considered.

A procedure is developed for determining feasible schedules in order to graduate a fixed number of trainees over time while minimizing the cost of facilities mix required.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Facilities Planning						
Course Scheduling						
Personnel Training						

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c.1 Course scheduling to find the minimum cost set of facilities required.